Logistic Modeling
Elaborate

Directions: Given the following scenarios plot the data on either DESMOS or a graphing calculator to see what model would fit the data best. Describe the scatter plots shape then create (by hand) a model for the data given the shape of the data and answer the given questions.

Problem 1:

The percentage of a corn crop (on a 200-acre farm) affected by a fungal blight over several weeks. The maximum percentage affected is 100%. The time, t, is measured in days.

a. Plot the data and explain the shape of the data. What function could model this data?

the graph looks like an s-shaped curve

function: logistic growth

b. Write the model for this data. Explain what the constants means within the context of the problem.

Time (Days)	Percentage of Crop Infected (%)
0	1
7	3
14	10
21	35
28	70 → close to critical
35	90
42	98
49	99.5

C: 100% → max % of corn infected
B: the initial starting condition (0,1) gives a B value of 99
k: there is a max growth rate close to 0.1944

B: $1 = \frac{100}{1+B} \Rightarrow 1+B = 100 \Rightarrow B = 99$

k: (28,70) → need to use close to critical point

$70 = \frac{100}{1+99e^{k(28)}}$

$\frac{70(1+99e^{-k(28)})}{70} = \frac{100}{70}$

$1+99e^{-k28} = \frac{100}{70}$

$\frac{99e^{-k28}}{99} = \frac{3}{7}/99$

$e^{-k28} = \frac{3}{7}/99$

$\frac{-k28}{-28} \ln = \frac{\ln(\frac{3}{7}/99)}{-28}$

$k = .1944$

$f(t) = \frac{100}{1+99e^{-0.1944t}}$

c. Compare your model to the data points. Are they exact? Justify your model?

*need to use (28,70) → close to critical point
if use (28,70) function is very close to data points
Not exact so there is some error

d. Use a graphing calculator to run logistic regression for the data points. Compare the equations to your model. If there are differences explain why there could be differences.

$f(t) = \frac{99.83}{1+156.34e^{-.2105t}}$; our function took into account 2 points, the regression took into account all points the error.

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Problem 2:

The following table gives the number of people infected with a new highly contagious virus on an isolated community of 1000 people.

a. Plot the data and explain the shape of the data. What function could model this data?

the data looks like an shaped curve, possible logistic growth

b. Write the model for this data. Explain what the constants means within the context of the problem.

Time (Days)	Number Infected
0	5
5	20
10	100
15	400
20	800 → close to critical
25	950
30	985
35	995

C: max population 1,000 → carrying capacity

B: initial growth condition for the virus is B = 199

k: exponential growth rate is k = 0.3340

B: $5 = \frac{1000}{1+B}$

$5(1+B) = 1000$

$1+B = 200$

$B = 199$

k: $800 = \frac{1000}{1+199e^{-k(20)}}$

$\frac{800(1+199e^{-k20})}{800} = \frac{1000}{800}$

$1+199e^{-k20} = \frac{5}{4}$

$\frac{199e^{-k20}}{199} = \frac{\frac{1}{4}}{199}$

$e^{-k20} = 0.001256$

$\frac{-k20}{-20} = \frac{\ln(0.001256)}{-20}$

$k = .3340$

$f(t) = \frac{1000}{1+199e^{-.3340(t)}}$

c. Compare your model to the data points. Are they exact? Justify your model?

The data points are close but there is error between the predicted and actual data points.

d. Use a graphing calculator to run logistic regression for the data points. Compare the equations to your model.

$f(t) = \frac{992.68}{1+324.29e^{-.3595t}}$; the model created by hand has the correct carrying capacity and initial condition. Though the regression used all data points to create the equation so it has minimal error.